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Annular Two-Phase Flow of Gases and Non-Newtonian Liquids

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Two-phase, annular flow of gases and non-Newtonian liquids through horizontal pipes is modeled, and a prediction method for pressure drop and gas volume fraction is proposed. The prediction method consists of a modification, based upon shear rate dependence of the apparent viscosity of the liquid phase, of Lockhart-Martinelli correlations for Newtonian liquids. The model applies to the case of laminar flow of the annular liquid film. Experimental data, obtained with several aqueous polymer solutions and air, agree closely with predicted values of pressure drop and void fraction.

SCOPE

The objective of this work is to develop techniques which enable prediction of pressure drop and gas volume fraction for two-phase, concurrent flow of gases and non-Newtonian liquids for the annular flow regime. This predictive capability is relevant to such processes as continuous polymerization reactors, direct contact drying of non-Newtonian liquid foods, etc.

Previous attempts to model two-phase, annular flow of gases and non-Newtonian liquids have included the assumption of a smooth gas-liquid interface. Partly as a result of this assumption, rather large discrepancies between predicted and experimental values of pressure drop and gas volume fraction have been observed. By basing a prediction scheme for non-Newtonian liquids on existing empirical correlations for Newtonian liquids, this assumption can be largely avoided.

The proposed prediction scheme accounts for the shear rate thinning behavior of pseudoplastic liquids. Since the average shear rate in the liquid phase is greater in the case of two-phase flow than in single-phase flow at the same flow rate, the average viscosity is taken into account quantitatively, using a power law fluid model. Effects directly attributable to the elasticity of the liquid are beyond the scope of this work.

Experimental data are presented for several types of aqueous polymer solutions at various concentration levels; these include three types in this work and three types used by previous investigators.

CONCLUSIONS AND SIGNIFICANCE

Based upon a comparison of predicted and measured pressure drops for annular, two-phase flow of gases and pseudoplastic fluids, the prediction scheme proposed here is quite successful. When the viscosity correction is not applied, the Lockhart-Martinelli correlation yields predicted values of pressure gradient which are approximately three times the measured values; however, when the correction is applied, the average discrepancy using the proposed prediction scheme is roughly 5% for all the solutions except Polyox. For the Polyox solutions, the predicted pressure drop is 10 to 40% less than the measured value; such disagreement appears to be attributable to the inadequacy of a simple power law fluid model to describe the rheological response of this type of solution in such two-phase flows.

Similarly, predicted values for the gas volume fraction agree, within about 10%, with measured values. For the results obtained in this laboratory, the measured volume fractions are slightly lower than predicted, whereas for the data obtained by previous investigators, the measured values are slightly higher.

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Based upon extensive experimental and theoretical studies of concurrent gas-liquid flows, one can readily predict pressure drop and void fraction for such flows for Newtonian liquids. According to DeGance and Atherton (1970), correlations such as those developed by Lockhart and Martinelli (1949) or by Dukler et al. (1964*b*) can be expected to yield predictions of pressure drop accurate to within 31% (standard deviation) and of void fraction accurate to within 18%. Moreover, flow regime maps, such as one with dimensionless coordinates recently developed by Taitel and Dukler (1976) for horizontal flow, enable one to predict the particular flow regime.

These correlations, however, are applicable only to gases and Newtonian liquids; for non-Newtonian liquids, our predictive capability is much weaker. Oliver and Young-Hoon (1968) measured pressure drop and liquid holdup for systems of air and aqueous polymer solutions and found that the experimental values for both pressure drop and liquid holdup were consistently lower than values predicted from the Lockhart-Martinelli correlation. They developed a model for annular flow which included turbulent gas flow in the core, a smooth gas-liquid interface, and a Newtonian liquid in laminar flow in the annular region. Using this model, their predicted mass flow rates differed by factors of 3 to 30 from the experimental values. Mahalingam and Valle (1972) conducted similar experiments and extended the Oliver and Young-Hoon model by taking into account the shear thinning characteristics of the pseudoplastic polymer solutions. However, their experimental values for pressure drop were ten to twenty times higher than their predicted values. Tyagi and Srivastava (1976), in work similar to that of Mahalingam and Valle, obtained ratios of experimental to predicted pressure drop as large as four.

All the models proposed above include the assumption of a smooth gas-liquid interface. Photographs of annular flows obtained by Hewitt and Hall-Taylor (1970) demonstrate that the interface is not smooth but wavy and irregular. Compared to the smooth interface model, then, the actual case of an irregular interface is one in which the average intensity of turbulence in the gas core is higher; thus, the actual pressure drop can be expected to be greater than that predicted by models incorporating the assumption of a smooth interface. Moreover, the models assume homogeneous phases, with no entrainment of one phase into the other; such entrainment might also contribute to greater pressure drop.

These considerations provide motivation for the exclusion of assumptions of smooth interface and no entrainment in the development of predictive equations for two-phase flow of gases and non-Newtonian liquids. The present work was undertaken to develop such equations, which would enable prediction of both pressure gradient and void fraction, given flow rates, fluid properties, and system geometry. In this work, only nonlinear viscous behavior of the liquid is taken into account. Although most non-Newtonian liquids also exhibit time dependent or elastic response, such behavior was not considered in the development of the model.

MODEL

In formulating a model to describe two-phase flow systems, one may choose one of two routes: the first entails an attempt to obtain analytic solutions to the coupled equations of motion for each phase, and the second incorporates the use of some empirical expressions to correlate experimental data. Although the analytic solution route appears more attractive, it forces us to make simplifying assumptions which may be so restrictive that the final solution lacks utility. For example, for the case of annular flow, including assumptions of smooth gas-liquid interface and no liquid entrainment in the gas core, one may readily obtain analytic solutions for the pressure gradient.

In order to appreciate the significance of these assumptions, we can compare the predictions of such an analytic solution with those obtained from an empirical expression, for example, the correlation of Lockhart-Martinelli (1949). Since the L-M correlation is based upon experimental data, it must take into account irregular interfaces and liquid droplet entrainment in the gas core, and thus we would expect the predicted pressure gradient to be higher for this case. For identical flow rates and Newtonian liquids, the empirical L-M correlation predicts pressure gradients two to three times larger than those predicted by the analytic solution.

Comparisons of this sort for Newtonian liquids provide strong motivation to include in the development of predictive equations for non-Newtonian liquids some means to account for the probable existence of a rough gas-liquid interface. The irregular nature of the interface dictates that this accounting be done on an empirical basis. By using an existing correlation for Newtonian fluids, the rough interface condition can be accounted for, and by modifying this correlation systematically to account for shear rate thinning behavior of the liquid phase, predictive equations for non-Newtonian liquids can be developed. This approach, if proven valid, affords greater confidence in the generality of the resulting equations, since it does not introduce any new empirical correlation.

In order to demonstrate the procedure whereby annular two-phase flow correlations for Newtonian fluids can be modified for non-Newtonian liquids, we have chosen the Lockhart-Martinelli correlations. Correlations for both pressure gradient and void fraction are required; both correlations are modified by the shear thinning characteristics of the non-Newtonian liquid. Further, the apparent viscosity of the fluid is assumed to be represented by a typical power law fluid model

$$\eta = K|\dot{\gamma}|^{n-1} \tag{1}$$

Since most polymer melts and solutions exhibit pseudoplastic behavior, the remarks below apply to this case, that is, $n < 1$.

The L-M pressure gradient correlation includes so-called two-phase multipliers $\phi_i(X)$ which are defined according to

$$\left.\frac{dp}{dz}\right|_{tp} = \phi_i^2 \left.\frac{dp}{dz}\right|_{si} \qquad i = L \text{ or } G \tag{2}$$

where $dp/dz|_{tp}$ is the pressure gradient for two-phase flow and $dp/dz|_{si}$ is the single-phase pressure gradient of phase i (at the same flow rate as in two-phase flow). ϕ_i depends on the character of the flow of each phase (laminar or turbulent) and is correlated by

$$\phi_i = \phi_i(X) \tag{3}$$

where

$$X = \left[\frac{\left.\dfrac{dp}{dz}\right|_{SL}}{\left.\dfrac{dp}{dz}\right|_{SG}} \right]^{1/2} \tag{4}$$

For two-phase flow, overall continuity considerations imply that the average shear rate in the liquid phase will be higher than in single-phase flow at the same liquid phase flow rate. Consequently, the average viscosity for the liquid phase will be lower in the two-phase flow case. This lower effective viscosity implies that the quantity $(dp/dz|_{SL})$ in Equation (4) will be too large. Since

$$\left.\frac{dp}{dz}\right|_{SL} = \frac{2}{R}\,\eta_{\text{wall}}\left(\frac{3n+1}{4n}\right)\left(\frac{4Q_L}{\pi R^3}\right) \tag{5}$$

it is appropriate to correct the estimate of the single-phase (liquid) pressure gradient by applying a factor equal to the ratio of the two apparent viscosities at the wall, namely

$$\left.\frac{dp}{dz}\right|^{*}_{SL} = \frac{\eta_{\text{wall}}|_{tpL}}{\eta_{\text{wall}}|_{SL}}\left(\left.\frac{dp}{dz}\right|_{SL}\right) \tag{6}$$

The problem then becomes one of evaluating $\eta_{\text{wall}}|_{tpL}$. This, in view of Equation (1), implies the need to evaluate the average shear rate at the wall $\dot{\gamma}_w|_{tp}$, where the average is considered to be taken over both time and length of pipe. Even for the case of annular flow with a rough interface, the time smoothed shear stress $\bar{\tau}_{rz}$ increases linearly with r over both the gas core and liquid annulus, and the stress is continuous over the interface. In addition, the average cross-sectional area through which the liquid flows is, on the average, that annular region extending from $\sqrt{\alpha}\,R$ to R, where α is the average gas volume fraction over the length of the pipe. The model used to derive $\dot{\gamma}_w|_{tp}$ is thus one in which, although temporal fluctuations occur in the gas-liquid interface location, these fluctuations contribute only to the roughness of the interface, and the location is constant, on the average, at $\sqrt{\alpha}\,R$. Similarly, in an average over both time and pipe length, the liquid phase undergoes rectilinear flow in the annular region from $\sqrt{\alpha}\,R$ to R. Axial symmetry is assumed; this was confirmed qualitatively by visual observations. Using this model, it can be readily shown that the average shear rate at the wall in the case of annular, two-phase flow is

$$\gamma_w\big|_{tp} = \frac{Q_L(n+1)}{\pi R^3 n}\left[1-\alpha-\left(\frac{2n}{3n+1}\right)\left(1-\alpha^{\frac{3n+1}{2n}}\right)\right]^{-1} \tag{7}$$

Note that Equation (7) applies to the case in which the annular liquid film is in laminar motion (rectilinear flow). Using Equation (7) for the two cases, $\alpha = \alpha$ and $\alpha = 0$ (single-phase flow), and Equation (1), one obtains

$$\frac{\pi_{\text{wall } tpL}}{\pi_{\text{wall } SL}} = \left\{\frac{n+1}{(3n+1)\left[1-\alpha-\left(\frac{2n}{3n+1}\right)\left(1-\alpha^{\frac{3n+1}{2n}}\right)\right]}\right\}^{n-1} \tag{8}$$

Therefore, to obtain the corrected form of the single-phase pressure gradient (liquid), only α, the gas volume fraction, and the power law exponent n are required. Since α cannot be determined a priori, we must resort to some predictive correlations. To be consistent, we can use the Lockhart-Martinelli liquid volume fraction correlation $\epsilon_L(X)$, where, since $\alpha = 1 - \epsilon_L$

$$\epsilon_L(X^*) = 1 - \alpha(X^*) \tag{9}$$

TABLE 1. POWER LAW PARAMETERS OF TEST LIQUIDS

Fluid	n (—)	K ($N \cdot s^n/\text{m}^2$)
70 wt % Glycerin	1.00	0.0143
1.5 wt % Separan AP30	0.340	8.96
0.75 wt % Separan AP30	0.429	1.47
0.15 wt % Separan AP30	0.523	0.198
0.75 wt % CMC-7H	0.532	1.13
0.39 wt % CMC-7H	0.602	0.228
1.0 wt % Polyox WSR 301	0.429	1.58
0.50 wt % Polyox WSR 301	0.603	0.166

In the present case, however, it is consistent to redefine X as

$$X^* = \left[\frac{-dp/dz|^{*}_{SL}}{-dp/dz|_{SG}}\right]^{1/2} \tag{10}$$

to account again for the shear thinning characteristics of the liquid. Since X^* is a function of α, Equations (6) and (8), the relationship $\alpha(X^*)$ is an implicit one. For a specific design case, however, solving for α is a straightforward procedure, requiring only four or five iterations on a hand calculator; for purposes of data analysis, we have developed a simple computer algorithm to solve for α.

By way of summary, the specific steps involved in the use of the present model include:

1. Solve, in order, Equations (8), (6), (10), and (9), using some initial guess for α, in an iterative manner until we obtain convergence for α and X^*. Required input data are simply flow rates, tube diameter, and fluid properties.

2. Calculate $-dp/dz|_{tp}$ from the L-M correlation for $\phi_L(X^*)$:

$$-\left.\frac{dp}{dz}\right|_{tp} = \phi_L^2\left(-\left.\frac{dp}{dz}\right|^{*}_{SL}\right) \tag{11}$$

In this model, the L-M correlation for viscous-liquid motion is used.

Implicit in the approximations

$$\phi(X^*) = \phi(X) \tag{12}$$

and

$$\alpha(X^*) = \alpha(X) \tag{13}$$

is the assumption that the viscous dissipation at the interface (alternatively, the average interfacial shear stress) for Newtonian liquids is identical to that for non-Newtonian liquids with equivalent shear viscosity. Such an assumption neglects elastic effects and the fact that the average shear stress at the interface is lower, and thus the average viscosity there higher, than at the wall. Further, the motions at the interface could include elongational as well as simple shearing flows; for complex solutions, the rheological response may differ markedly in the two flows (Weinberger and Goddard, 1974). However, these approximations are necessitated at the outset by the desire to avoid the introduction of a new and separate empirical correlation for non-Newtonian liquids. This advantage tends to outweigh the disadvantage of introducing an assumption whose accuracy cannot be calculated a priori. Moreover, incorporation of the assumption provides at least one basis with which to compare interfacial shear behavior of Newtonian fluids with that of non-Newtonian fluids.

Another assumption implicit in the current estimation procedure is that elastic behavior of the non-Newtonian liquids is negligible in comparison to the pseudoplastic,

TABLE 2. ADDITIONAL TESTS OF PRESSURE GRADIENT PREDICTION

				dP/dz (kN/m^3)	
Fluid	Q_L (cm^3/s)	Q_G (cm^3/s)	D^* (mm)	Measured	Predicted
0.35% CMC	2.95	157	2.90	77.8	66.9
0.75% CMC	3.05	157	2.90	124	135
0.50% Polyox	3.16	157	2.90	107	54.1
1.0% Polyox	2.78	157	2.90	147	79.0
0.65% Methocel	15.8	944	12.7	0.204†	0.446
0.50% Methocel	15.8	944	12.7	0.401	0.410
0.25% Methocel	15.8	944	12.7	0.190	0.163
1.50% SCMC	5.43	236	6.35	3.07	3.78
1.50% SCMC	5.43	393	6.35	4.96	4.38
1.50% SCMC	10.1	236	6.35	4.96	4.89
1.30% Polyox	5.43	236	6.35	2.92	2.56
1.30% Polyox	5.43	393	6.35	3.95	3.00
1.30% Polyox	10.1	236	6.35	4.53	3.30
1.30% Polyox	10.1	393	6.35	5.99	3.84

* Tube diameter provides key to source of data: 2.90 mm—this laboratory, 12.7 mm—Mahalingam and Valle (1972), 6.35 mm—Oliver and Young-Hoon (1968).

† Datum apparently in error. Note measured pressure gradient at lower concentrations.

or shear thinning, behavior. Although heuristic arguments, based upon the rectilinear flow of such liquids through ducts with a high ratio of length to diameter, can be offered to support this assumption, its use also represents a concession to the extreme difficulty of incorporating more realistic fluid models which include some description of elastic response.

All of these considerations suggest potential limitations to the proposed model and areas of possible improvement. The first step, though, is to compare predictions obtained from the model with experimental data; these comparisons are described below.

EXPERIMENTAL

Aqueous solutions of Separan AP30, a polyacrylamide with a molecular weight of 2 to 3 million, manufactured by the Dow Chemical Co., were used as the non-Newtonian test solutions. Polymer solutions were used so that the experiments could be performed at ambient temperature. Three solutions, with concentrations ranging from 0.15 to 1.5 wt %, were used. The solutions were used on a once through basis in order to avoid any shear induced degradation.

The shear viscosity functions for each solution were determined by using the experimental apparatus as a capillary viscometer. This permitted the apparent viscosity to be measured at shear rates close to those occurring during two-phase flow. Since the L/D of the tube was so large, approximately 600, end corrections did not have to be made.

The solutions closely exhibited power law behavior, and the single-phase flow data were analyzed by standard techniques (Middleman, 1968) to obtain values of the flow consistency K and the power law exponent n. All of the solutions exhibited pseudoplastic behavior (Table 1). Primarily in order to assure ourselves that the system was operating satisfactorily, a Newtonian fluid, a 70 wt % glycerin solution, was also tested; its viscosity is also listed in Table 1.

The experimental apparatus is shown schematically in Figure 1. A 1 gal stainless steel tank supplied liquid to a variable speed MOYNO pump (type CDO) with a discharge capacity of 1.5 to 4.2 cm^3/s. The pump discharged to a T type of mixer with a 0.794 mm orifice on the liquid arm. The narrow orifice prevented pulsation of the liquid entering the tube.

Bottled industrial dry air was introduced via a rotameter (50 to 300 cm^3/s @ 1 atm and 25°C) to the other arm of the mixer. The air-liquid mixture then passed through a 2.9 mm (ID), heavy wall, horizontal, Plexiglas tube, 1.72 m long.

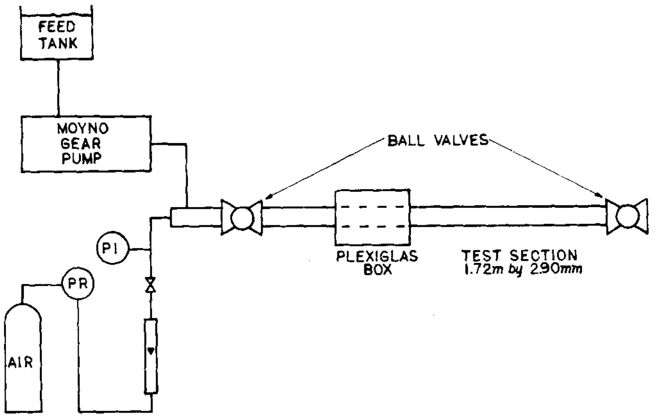

Fig. 1. Sketch of apparatus.

The pressure drop across the test section was measured at a point slightly upstream of the mixer, and the gas pressure drop between the pressure tap and the mixer was found to be negligible. The tube discharged to the atmosphere. Pressure in the range of 0 to 103 kN/m^2 (0 to 15 lb/in.^{2} gauge) was measured using a HEISE gauge with a 380 mm face (sensitive to 0.07 kN/m^2). Pressure in the range of 0 to 414 kN/m^2 (0 to 60 lb/in.^{2} gauge) was measured using a Maxisafe test gauge with a 150 mm face (sensitive to 0.07 kN/m^2). Both of these gauges are Bourdon tube gauges and were calibrated using a mercury manometer.

Visual observation of the flow was facilitated by fitting the tube with a Plexiglas box filled with water to reduce the distortions caused by the curvature of the tube. These observations assured us that our measurements were obtained in the annular flow regime and that the flow was axisymmetric.

The test section was fitted with two ball valves with mechanical linkage between the two valves which enabled them to be closed simultaneously. The ball valves were placed in the apparatus only for a few average liquid volume fraction measurements.

The procedure for pressure drop experiments was extremely simple. The liquid and gas were simply combined in the mixer and allowed to flow concurrently in the tube. The only problem which occurred was liquid backing up into the gas line and pressure tap line. By starting the gas flow first, the liquid was prevented from entering the gas line and the pressure tap line. To insure further that no liquid entered the gas lines, the first data were taken at the highest liquid flow rate, and

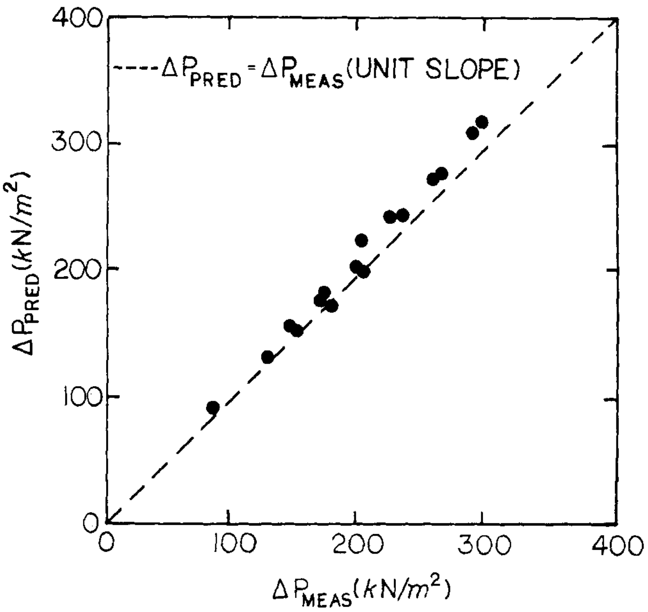

Fig. 2. Comparison of predicted and measured pressure drops, 70% glycerin-water.

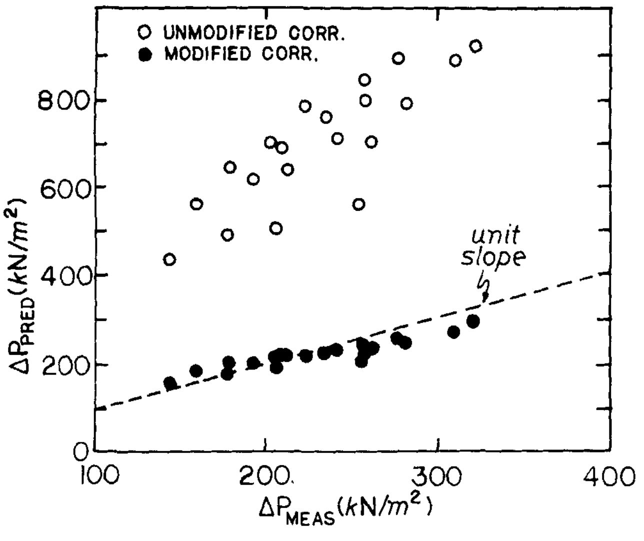

Fig. 3. Comparison of predicted and measured pressure drops, 1.5% Separan AP-30.

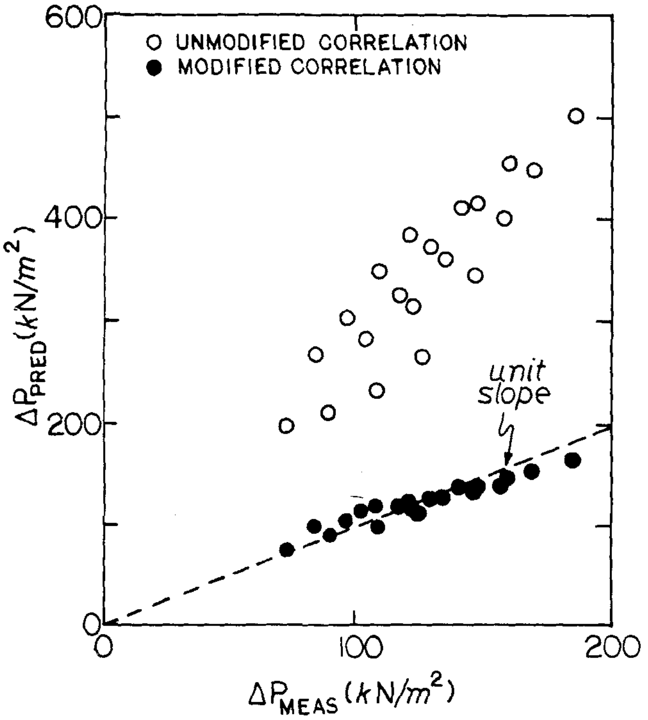

Fig. 4. Comparison of predicted and measured pressure drops, 0.75% Separan AP-30.

the liquid flow rate was reduced for subsequent data. This allowed a reduction in system pressure with a change in the liquid flow rate, and thus some air would bleed from the air lines and no liquid would enter.

Twenty data points were taken for each concentration of Separan AP-30 (five gas flow rates at each of four liquid flow rates). The gas flow rate was measured using a rotameter, and the liquid flow rate was determined by weighing a 1 min sample. The liquid pressure drop (with no gas flowing) was measured at each flow rate and the results used to determine the shear flow characteristics (for a power law model) for each liquid tested. All tests were made at 28°C. Tabulated data, along with a more detailed description of the apparatus, are available elsewhere (Eisenberg, 1977).

RESULTS AND DISCUSSION

The results of the pressure drop experiments are included in Figures 2 through 5. In each plot, predicted values of pressure drops from both the modified and unmodified (no viscosity correction) Lockhart-Martinelli correlations are plotted vs. the measured pressure drop. The unit slope line indicates where the predicted pressure drop, using the modified correlations, would exactly equal the measured pressure drop. In all cases, the liquid-viscous, gas-turbulent correlation of L-M is used.

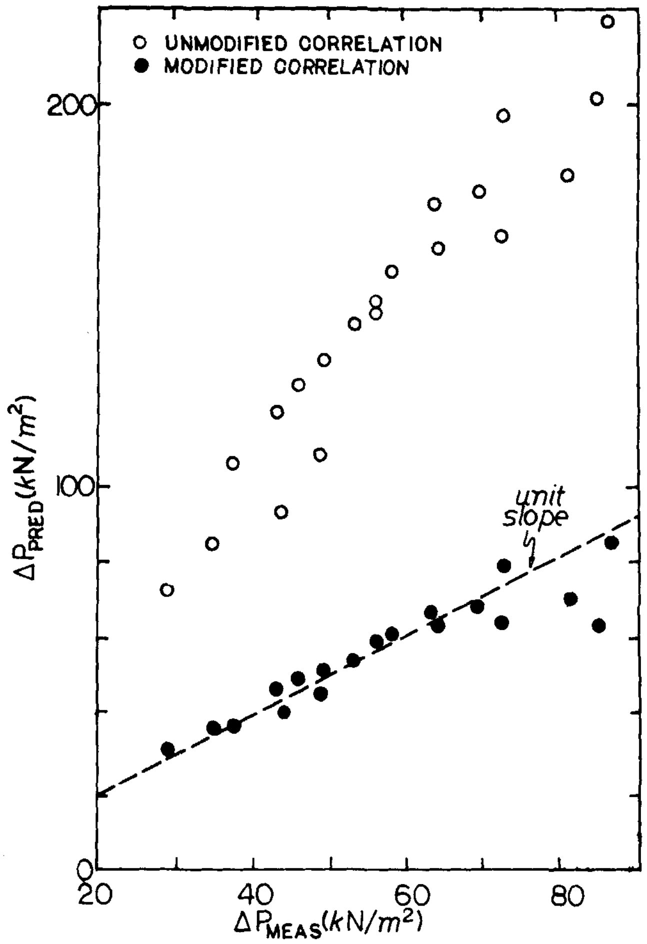

Fig. 5. Comparison of predicted and measured pressure drops, 0.15% Separan AP-30.

Figure 2 compares the predicted and measured values of pressure drop for the 70 wt% glycerin solution. For this Newtonian liquid, the predicted pressure drop compares extremely well to the measured pressure drop (±10%) over a threefold range of this variable, thus confirming the applicability of the Lockhart-Martinelli correlation for this experimental apparatus. Such good agreement also provides strong support for the use of a model which is a modified form of the Lockhart-Martinelli correlation (as opposed to some other correlation) to correlate our data for non-Newtonian liquids.

Corresponding comparisons for the aqueous polymer solutions are provided in Figures 3 through 5. Identical gas and liquid flow rates were used in each of the figures. The gas flow ranged from 70 to 200 cm^3/s, and the liquid flow ranged from 1.58 to 4.12 cm^3/s. As might be expected, the predicted pressure drops obtained from the unmodified Lockhart-Martinelli correlations are significantly larger (factors of two to three) than the measured pressured drops. However, using the proposed model in conjunction with the L-M correlation yields agreement within 20%. Such agreement is even better than the average deviation expected for Newtonian liquids (Dukler et al., (1964*a*). The method, therefore, appears to represent a significant improvement over previous

Table 3. Test of Gas Void Fraction Prediction

Fluid*	Q_L (cm^3/s)	Q_G (cm^3/s)	Gas void-fraction, α Measured	 Predicted
0.35% CMC	4.23	70	0.60	0.62
0.35% CMC	1.60	200	0.78	0.79
0.50% Polyox	4.20	70	0.57	0.63
0.50% Polyox	1.60	70	0.60	0.67
0.65% Methocel	15.8	944	0.75	0.62
0.50% Methocel	15.8	944	0.75	0.64
0.25% Methocel	15.8	944	0.84	0.72
1.5% SCMC	5.43	393	0.64	0.58
1.3% Polyox	5.43	393	0.72	0.62

* See key for Table 2 for source of data.

attempts to predict pressure gradients for two-phase, annular flow of gases and non-Newtonian liquids. In addition, no new empirical correlation is involved in this method.

Experimental confirmation of the model using only three polymer solutions and a single tube diameter may be regarded as inadequate, however. Accordingly, the data of Mahalingam and Valle (1972), hereafter M-V, and Oliver and Young Hoon (1968), hereafter O-H, were analyzed according to our model. The analysis involved some uncertainty, since the flow regime was delineated only approximately in their papers. Further, additional tests were conducted in our laboratory with two aqueous carboxymethylcellulose (CMC) solutions and two aqueous polyethylene oxide (Polyox) solutions. The results are summarized in Table 2. Except for the Polyox solutions, most of the predicted pressure gradients agree within 10% with the measured values. (The measured pressure gradient for the 0.65% methocel solution appears to be in error, since it is less than that for the 0.5% solution at the same flow rates.) This agreement, extending over a hundredfold range of pressure gradients and a fourfold range of tube diameter, provides significant corroboration of the proposed prediction scheme.

For all of the Polyox solutions, the prediction scheme underpredicts the pressure gradient by as much as a factor of two. Moreover, as shown in Table 2, the discrepancy is greater with our data than with those of O-H. The discrepancy for this particular solution, though less than that obtained with previous prediction schemes, is far greater than observed for the other polymer solutions. This suggests that the rheological response of the Polyox solution is significantly different from that of the other solutions. Since the power law fluid model we employ is a purely viscous model, liquids which are highly elastic or which exhibit large tensile stresses in stretching motions may not be adequately described by our prediction scheme. Although Separan and Polyox solutions are both known to exhibit elastic behavior (Huppler, Ashare, and Holmes, 1967), Polyox solutions tend to develop high tensile stresses more rapidly in stretching motions than do Separan solutions (Weinberger and Goddard, 1974). Associating differences in dynamic response in two-phase flow to a specific liquid rheological property would be premature, however. Establishing the precise boundaries of validity of the present prediction method with respect to fluid properties requires extensive material characterization and additional testing with other liquid systems; accordingly, this must be left for future work. Of main interest at this point is the fact that the proposed prediction scheme works very well for a number of viscoelastic polymer solutions and does less well for one polymer solution which exhibits significant elastic response.

Although the proposed model was derived in order to predict annular two-phase pressure gradient, a convenient by-product of this model is the concomitant prediction of the average gas volume fraction. Several experiments were conducted to measure gas volume fractions and to compare these with predicted values.

The comparisons, for data obtained in our laboratory and also by M-V and O-H are given in Table 3. It can be seen that our measured values for α are slightly greater than the predicted values for α, whereas the converse is true for the data of M-V and O-H. However, with the exception of the 0.65% methocel datum, the discrepancy for each data point is within the 16.3% average discrepancy computed by Dukler et al. (1964*a*) from experimental data for Newtonian liquids. Thus it appears that the proposed model successfully predicts average gas volume fraction as well as pressure gradient.

NOTATION

D = tube diameter
K = consistency index
n = power-law exponent
p = pressure
Q = volumetric flow rate
R = tube radius
v_z = axial velocity
X = Martinelli parameter, defined by Equations (4) and (16)
z = axial coordinate

Greek Letters

α = average gas volume fraction
$\dot{\gamma}$ = shear rate
ϵ_L = average liquid phase volume fraction
η = apparent shear viscosity of liquid phase
τ = shear stress
ϕ_i = Martinelli parameter for phase i, Equations (2) and (3)

Subscripts

G = gas phase
i = phase subscript, L or G
L = liquid phase
s = single phase
tp = two phase
w = wall

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Enhancement of Heat Transfer and Thermal Homogenity With Motionless Mixers

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Motionless mixers are well suited to augment heat transfer and to provide a uniform thermal environment in laminar flow reactors. Heat transfer performance can be characterized with the Nusselt number, while the degree of thermal homogeneity can be characterized using the thermal time distribution which is the nonisothermal analogue of the residence time distribution. Theoretical results, calculations, and design strategies are presented for a variety of mixer types including flow inverters as well as the more common flow dividers. Two-channel, partial flow inversion is an excellent, near optimal, strategy for cooling or other situations such as polymerizations which give elongated velocity profiles. For flattened velocity profiles, perfect mixing can sometimes give better results. In practice, however, partial inverters can be expected to outperform those motionless mixers which are primarily flow dividers.

SCOPE

Laminar flow heat exchange is characterized by large temperature gradients within the fluid and by rather low heat transfer coefficients. In reacting systems, this nonuniform environment can give rise to large concentration gradients. A standard means of increasing heat transfer and of providing more uniform reaction conditions is to use multitube or extended surface heat exchangers, but these are expensive and may give rise to flow instabilities, particularly for polymerizations. An alternate, more recently emerging technology is to use motionless mixers installed within the flow duct. These are static devices which use the energy of the flowing fluid to promote mixing throughout the cross section of the duct. A typical device divides and recombines the flow stream, reducing the striation thickness by a factor of two or more for each elemental unit of the mixer. A series combination of such elements can approach homogeneity on a molecular scale, but even these nearly perfect mixers do not provide the best means for enhancing heat transfer. Instead, relatively simple devices known as a flow inverters maximize the driving force for heat transfer by interchanging material between the centerline and the walls.

When the length of the motionless mixer is short compared to the total length of the heat exchanger, heat transfer performance can be characterized in some detail. If the velocity profile is independent of axial position, that is, if fluid properties are nearly constant, rather powerful analytical techniques give optimal design parameters and optimal mixer locations for arbitrary flow geometries and fluid rheologies. If fluid properties are temperature sensitive or if highly accurate estimates of the Nusselt number are desired, numerical calculations are necessary, but a considerable degree of generalization and a reasonably robust design strategy remain possible. This strategy offers significant improvements in performance without lengthy optimization studies.

CONCLUSIONS AND SIGNIFICANCE

Commercial motionless mixers are difficult to analyze theoretically owing to complicated three-dimensional flows and the accompanying lack of symmetry. This paper takes a first step toward rigorous analysis by dividing the overall mixing process into simpler components of flow inversion and symmetry preserving flow division. The analysis shows an inherent weakness of flow division for augmentation of heat transfer coefficients. Better, nearly optimal results can be achieved with a practical, low pressure drop device known as the two-channel partial flow inverter. For parabolic and elongated velocity profiles, this device has the same asymptotic performance as a perfect mixer and outperforms a perfect mixer for all intermediate values of the Graetz parameter. For Graetz parameters above about 10 (Graetz numbers above 30), a single flow inverter installed midway in a heat exchanger will give a 25 to 30% improvement in Nusselt number. Larger enhancements in Nu are possible in a multizone heat exchanger with two or more flow inverters, but for very long heat exchangers, the best practical strategy is to intersperse an occasional motionless mixer of the conventional flow division type.

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